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NUMERICAL STUDY OF SWIRLING ONE- AND TWO-PHASE TURBULENT FLOWS IN A CYLINDRICAL CHANNEL

V. V. Novomlinskii and M. P. Strongin

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Turbulent swirling flows are widely used to intensify heat and mass transfer processes in different types of processing units. Examples of the latter are plasma-chemical reactors plasmatrons, combustion chambers, scrubbers, etc. To make these units more efficient, it is necessary to make a detailed study of the hydrodynamics in swirling flows.

It is known that swirling flows are characterized by highly curved streamlines and the development of recirculation zones. The location and dimensions of these zones depend to a considerable extent on the intensity of swirling and the configuration of the boundaries of the flow. The dimensions of the recirculation zones also depend on the "charging" of the flow with particles in the case of dispersed-gas flows. The study of vortical flows with a disperse phase is complicated by the need to allow for dynamic interaction of the phases. This, together with the problem of modeling the turbulence, makes it more difficult to numerically study such flows. The theoretical and experimental investigation of swirling flows was given great impetus in [1-3].

1. Swirling Turbulent One-Phase Flows. The large amount of interest in intensive swirling flows – the main type of turbulent flow – requires the use of fairly flexible turbulence models. The study [4] presented the results of calculations of axisymmetric swirling turbulent jets using the Prandtl mixing length model. The results agreed well with experimental findings. In [5, 6] an attempt was made to use the standard $k-\epsilon$ model of turbulence to numerically study swirling flows (k is the kinetic energy of the pulsating motion and ϵ is the rate of dissipation of pulsative energy). This model has proven to be useful in calculations of simple shear flows. However, use of the standard $k-\epsilon$ model in the case of fairly intensive swirling has led to a significant deviation from the experimental results. The authors of [6] explain this discrepancy by citing the anisotropy of eddy viscosity, although the standard turbulence model they used does not even take into account the expressions for the fluctuation moments which appear due to swirling and make a description possible within the framework of an isotropic model. It was noted in [7] that one way of further improving turbulence models for swirling flows is modifying the $k-\epsilon$ model in different ways.

In [8-12], corrections were proposed for the traditional two-parameter model. As noted in [12], all of the modifications proposed earlier for the $k-\epsilon$ model proved unsuitable for calculating bounded swirling flows. The approach taken by the authors of [12] consisted of selecting optimum values of the empirical constants of the energy–dissipation model to study

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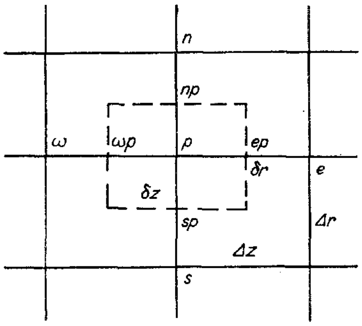

Fig. 1

bounded swirling turbulent flows. Thus, by direct selection and comparison with the experiment in [13], the authors obtained the following optimum values for the constants: $c_\mu = 0.125$, $c_1 = 1.44$, $c_2 = 1.8943$, $\sigma_\varepsilon = 1.1949$. However, as was noted in [12], these optimum values are unacceptable for calculations of nonswirling flows. It is proposed in [12] that nonswirling flows be calculated using a single set of empirical constants, while another set is to be used to calculate slightly or intensively swirled flows. However, it is obvious that the characteristics of unswirled and slightly swirled flows should be similar and should differ appreciably from the parameters of an intensively swirled flow.

Another approach to modeling turbulence for swirling flows consists of using a second-order model which includes equations for the transport of second one-point moments $\langle u_i' u_j' \rangle$. This made it possible to eliminate the hypothesis that eddy viscosity is isotropic [14, 16]. Unfortunately, these models have a serious deficiency: the presence of a large number of empirical constants. The latter seriously diminishes the practical utility of the model. Also, the additional equations for $\langle u_i' u_j' \rangle$. make the model more cumbersome, which makes its numerical realization more difficult. The validity of the hypotheses used in deriving these models [17] remains an open question. Attempts to use the second-order model to calculate swirling flows have proven to be "very disappointing" [17].

Here, we propose a new modification of the k–ε model for swirling flows of constant density. The new model considers the effect of swirling on the turbulence characteristics of the flow, and it changes into the standard model when swirling decays. Almost all of the previous modifications of the k–ε model involved a change in the expressions for the empirical constant c_2 and/or c_μ. Here, we attempt to modify the term in the ε-equation connected with generation so as to empirically account for additional correlations which occur in swirling flow. The better mixing of swirling flows is due to their higher degree of turbulence compared to nonswirling flows, i.e. to their larger transfer coefficients. In using a closed k–ε model, where $\mu_t = c_\mu \rho k^2/\epsilon$ is eddy viscosity, we can attempt to obtain an increase in μ_t in the swirling flow by modifying the equation for ε through a change in the term with generation in the ε-equation – since this term has introduced into the equation with a high degree of arbitrariness. Any modification should of course deal with the generation of turbulence due to swirling, since it is necessary for sufficiently flexible turbulence models that they revert to the standard k–ε model as the intensity of swirling decays at the limit and approaches zero. In certain cases, swirling of a turbulent flow leads to suppression of turbulence and, thus, to a reduction in the eddy viscosity coefficient. For example, the author of [18] presented a criterion of the decay of turbulent pulsations under the influence of a radial force in a swirling flow $\partial(w^2/r)/\partial r > 0$.

In fact, in the description of a turbulent swirling flow on the basis of the k–ε model, a new moment $\langle F_r' v' \rangle$ appears. Here F_r' is the fluctuational component of the radial (centrifugal) force. In accordance with the gradient hypothesis, this moment can be represented in the form (incompressible case) $\langle F_r' v' \rangle = -\mu_t \partial(w^2/r)/\partial r$. Writing the generation term G as the sum of G_w and $G_{u,v}$ (G_w is the generation due to the tangential component of velocity w and $G_{u,v}$ is the generation due to the components u, v), we obtain $G_w = \mu_t \left[\left(r \frac{\partial}{\partial r} \left(\frac{w}{r} \right) \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 - \frac{\partial (w^2/r)}{\partial r} \right]$.
An increase (decrease) in μ_t is obtained by decreasing (increasing) the term with generation in the equation for ε. Using a simple analog for the Richardson number $\mathrm{Ri} = G_w^s / (G_{u,v}^s + G_w^s)$, we can assign c_1 in the form $c_1 = 1.44 - c_3 \mathrm{Ri}$. Here, the superscript s denotes the shear part of generation without the term $\langle F_r' v' \rangle$. The best agreement with the experimental data [13] is obtained when $c_3 \approx 1$, i.e.

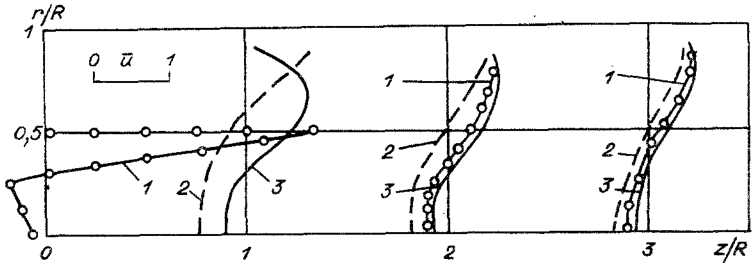

Fig. 2

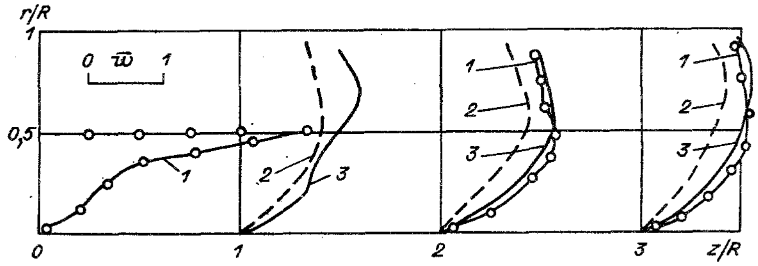

Fig. 3

$$c_1 = 1.44 - \mathrm{Ri}. \tag{1.1}$$

Assuming that there is axial symmetry, the steady turbulent swirling flow of an incompressible fluid in a cylindrical channel is described on the basis of the Reynolds equations closed by means of a modified turbulence model. The k–ε model is used with the modification proposed in [19] for boundary-layer flows. In a cylindrical coordinate system with the z axis along the channel axis and the radial coordinate r, the equations have the form

$$\frac{\partial \rho u r}{\partial z} + \frac{\partial \rho v r}{\partial r} = 0, \tag{1.2}$$

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial z} + 2\frac{\partial}{\partial z}\left(\mu_s \frac{\partial u}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_s \frac{\partial v}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_s \frac{\partial u}{\partial r}\right),$$

$$\rho u \frac{\partial v}{\partial z} + \rho v \frac{\partial v}{\partial r} - \frac{\rho w^2}{r} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial z}\left(\mu_s \frac{\partial v}{\partial z}\right) + \frac{2}{r}\frac{\partial}{\partial r}\left(r\mu_s \frac{\partial v}{\partial r}\right) +$$

$$+ \frac{\partial}{\partial z}\left(\mu_s \frac{\partial u}{\partial r}\right) - 2\mu_s \frac{v}{r^2},$$

$$\rho u \frac{\partial w}{\partial z} + \rho v \frac{\partial w}{\partial r} + \frac{\rho v w}{r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_s \frac{\partial w}{\partial r}\right) + \frac{\partial}{\partial z}\left(\mu_s \frac{\partial w}{\partial z}\right) - \frac{w\mu_s}{r^2} - \frac{w}{r}\frac{\partial \mu_s}{\partial r},$$

$$\rho u \frac{\partial k}{\partial z} + \rho v \frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\mu_s}{\sigma_k}\frac{\partial k}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\mu_s}{\sigma_k}\frac{\partial k}{\partial z}\right) + G - \rho\varepsilon,$$

$$\rho u \frac{\partial \varepsilon}{\partial z} + \rho v \frac{\partial \varepsilon}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\mu_s}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\mu_s}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z}\right) + c_1 f_1 \frac{\varepsilon}{k} G - c_2 f_2 \rho \frac{\varepsilon^2}{k},$$

$$c_1 = 1.44 - \mathrm{Ri},\ \mu_s = \mu_t + \mu_l,\ G = G_{u,v} + G_w,$$

$$\mathrm{Ri} = G_w/(G_{u,v} + G_w),\ \mu_t = c_\mu f_\mu \rho k^2/\varepsilon,$$

$$G_w = \mu_s\left[\left(r\frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 - \frac{\partial}{\partial r}\left(\frac{w^2}{r}\right)\right],$$

$$G_{u,v} = 2\mu_s\left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{v}{r}\right)^2\right) + \left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial z}\right)^2,$$

$$f_\mu = (1 - \exp(-A_\mu R_k))^2 (1 + A_t/R_t),$$

$$f_1 = 1 + (A_1/f_\mu)^3,\quad f_2 = 1 - \exp(-R_t^2),$$

$$R_t = k^2\rho/\mu_l\varepsilon,\quad R_k = k^{1/2}(R - r)\rho/\mu_l,$$

$$r = R:\ \mu_t = \frac{\partial k}{\partial r} = 0,\quad \varepsilon_w = \frac{\mu_l}{\rho}\left(\frac{\partial^2 k}{\partial r^2}\right)_w.$$

Here, u, v, and w are the z, r, and φ components of the averaged velocity, respectively; $k=\frac{1}{2}\langle u_i'u_i'\rangle$ is the kinetic turbulent energy; $\varepsilon=\nu_l\left\langle\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}\right\rangle$ is the specific rate of dissipation of fluctuation energy; ρ and p are the averaged density and pressure of the gas; μ_s, μ_l, μ_t . are the effective, laminar, and eddy viscosities. The empirical constants of the k–ϵ model take the standard values [19]: $c_\mu = 0.09$, $c_2 = 1.92, \sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $A_\mu = 0.0165$, $A_t = 20.5$, $A_1 = 0.05$. The following boundary conditions were adopted for problems concerning a turbulent swirling flow in a cylindrical tube: "mild" boundary conditions, with axial gradients equal to zero, are assigned at the tube outlet for all parameters; the profiles obtained experimentally in [13], with v = 0, are assigned at the inlet for u, w. The condition $\partial\Phi/\partial r = 0$ is satisfied on the tube axis for Φ = u, w, k, ϵ, and v = 0. Assigning a "stepped" rather than a uniform profile for k and ϵ on the inlet boundary made it possible to have the calculated results approximate the experimental data.

Any differential equation of system (1.2) can be represented in the form

$$\frac{\partial\rho u\Phi}{\partial z}+\frac{1}{r}\frac{\partial\rho vr\Phi}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\Phi\frac{\partial\Phi}{\partial r}\right)+\frac{\partial}{\partial z}\left(\Gamma_\Phi\frac{\partial\Phi}{\partial z}\right)+S_\Phi, \tag{1.3}$$

where Φ = u, v, w, k, ϵ; Γ_Φ are transport coefficients; S_Φ are source terms. Integrating (1.3) over the control volume of the difference grid (Fig. 1), we obtain its discrete analog [20]

$$\begin{aligned}
&a_p\Phi_p = a_e\Phi_e + a_\omega\Phi_\omega + a_n\Phi_n + a_s\Phi_s + b,\\
&a_e = D_eA(\mathrm{Pe}_e) + \max(-F_e,\ 0), \quad D_e = \Gamma_{ep}r_e\Delta r/\delta z,\\
&a_n = D_nA(\mathrm{Pe}_n) + \max(-F_n,\ 0), \quad D_n = \Gamma_{np}r_{np}\Delta z/\delta r,\\
&a_\omega = D_\omega A(\mathrm{Pe}_\omega) + \max(F_\omega,\ 0), \quad D_\omega = \Gamma_{\omega p}r_\omega\Delta r/\delta z,\\
&a_s = D_sA(\mathrm{Pe}_s) + \max(F_s,\ 0), \quad D_s = \Gamma_{sp}r_{sp}\Delta z/\delta r,\\
&F_e = (\rho u)_{ep}r_e\Delta r, \quad F_n = (\rho v)_{np}r_{np}\Delta z,\\
&F_\omega = (\rho u)_{\omega p}r_\omega\Delta r, \quad F_s = (\rho v)_{sp}\Delta z r_{sp},\\
&r_{sp} = (r_s + r_p)/2, \quad b = S_c\Delta z r\Delta r,\\
&a_p = a_e + a_\omega + a_n + a_s - S_p r_p\Delta r\Delta z.
\end{aligned}$$

The quantities Γ_{ep}, Γ_{np}, $\Gamma_{\omega p}$, Γ_{sp}, $(\rho v)_{sp}$, $(\rho u)_{ep}$, $(\rho u)_{\omega p}$, $(\rho v)_{np}$ are also determined, while S_c and S_p are found by linearizing the source term $S_\Phi = S_c + S_p\Phi_p$. The grid Peclet numbers Pe_e, Pe_ω, Pe_n, Pe_s are represented as the ratio of the corresponding convective and diffusive terms: $\mathrm{Pe}_\alpha = F_\alpha/D_\alpha$, $\alpha = e, \omega, n, s$. The expression for the difference coefficients a_e, a_ω, a_n, a_s contain the unknown function $A(\mathrm{Pe}_\alpha)$, which also determines the type of difference scheme. The author of [20] recommended a power scheme to model two- and three-dimensional problems. For this scheme, $A(\mathrm{Pe}_\alpha) = \max[0,\ (1-0.1|\mathrm{Pe}_\alpha|)^5]$.

In deriving the discrete analog for the equations of the velocity components u and v, it is convenient to shift the control volume to the right and upward from the point p, respectively (i.e. to use a so-called "staggered grid"). The values of u and v are found at the points ep and np. One consequence of the use of such a grid is that the pressure difference between two adjacent nodes determines the velocity component at the point between the nodes. Such a location of control volumes has other advantages as well [20]. The pressure field can be calculated by means of the SIMPLE-procedure [20]. The difference equations for all of the variables were solved by an iterative method using trial runs in the r direction.

To validate the model obtained here, we performed a series of calculations for a turbulent swirling flow of air in cylindrical channel. Figures 2 and 3 show the radial profiles of the dimensionless velocity components u and w. Comparison of the experimental data ln [13], the calculated data in [12], and the results of modeling by the method described above (curves 1-3, respectively) showed that the model with a correction for c_1 predicts the flow characteristics most accurately. It should be noted that in calculations with the standard k–ϵ model, a profile of w with a maximum near the wall is quickly established. This is characteristic of the rotation of a solid and is not in agreement with the experiment.

2. Turbulent Swirling Dispersed-Gas Flows. A limited number of studies have calculated swirling flows with a disperse phase. The study [21] employed analytical methods to investigate limiting cases of the laminar flow of a swirling dust-laden gas. The investigation [22] examine the motion of solid particles in a specified velocity field. The components of

the velocity vector of the gas phase were modeled by means of an arbitrary time function distributed according to a normal law with a mathematical expectation equal to the mean values of the velocities and a variance equal to the turbulence intensity. The studies [23, 24] presented calculations of swirling flows with a disperse phase on the basis of the equations of gas dynamics.

Here, we examine a turbulent swirling flow with a disperse phase when the Navier–Stokes equations are used for the carrier phase with a modified closed $k-\epsilon$ model. The hypothesis of interpenetrating, interacting continua was adopted to model two-phase flows. The disperse phase consists of spherical solid particles of the same size. The volume concentration of these particles is small, while their mass percentage may be substantial. Since the particle concentration is small, we did not consider the interaction of the particles in the flow. The effect of the particles on both the averaged and the fluctuation parameters of the flow was considered [25]. The study [26] showed that fluctuations of the parameters of the particles can be ignored for a certain class of flows, which simplifies the model considerably. In many flows containing heavy particles encountered in practice, $\tau_0/\tau_D < 1$, where τ_D is the dynamic relaxation time of a particle in the flow, $\tau_0 = L/u_0$ is the characteristic time of large-scale fluctuations of the dimension L, $\tau_0 \sim 10^{-3}$-10^{-4} sec, and $\tau_D \sim 10^{-2}$ sec. This allows us to ignore fluctuations in the parameters of the disperse phase for inert particles when examining phase interaction and modeling the motion of a two-phase medium. In swirling flows, the effect of turbulent transverse diffusion of particles is usually negligible compared to radial dispersion of particles due to manifestation of their velocity component w_p.

We used Eqs. (1.2) to describe the motion of the carrier phase, adding source terms reflecting the interaction of the gas and disperse phases (only drag is considered) to the right sides of the equations for u, v, w, k, and ϵ. The Friedman–Keller procedure is normally used to derive these terms in the equations for k and ϵ. The derivation is based on the assumption of continuity of the disperse phase for all characteristic dimensions of the problem, including the correlative dimension of ℓ for ϵ, i.e. on the microscale of the turbulent pulsations. However, as shown in [25], it is not possible to formally apply the Friedman–Keller procedure to obtain the term $S_{\epsilon p}$ – which considers the interaction of the phases in the equation for ϵ – because the disperse phase can no longer be regarded as a continuum in this case. The possibility of ignoring the term $S_{\epsilon p}$ in the equation for dissipation was substantiated for the case of satisfaction of the condition $\ell/r_0 << 1$ (r_0 is the mean distance between particles). The source terms for the other equations have the form

$$S_{up} = \theta(u_p - u)/\tau_D,\ S_{vp} = \theta(v_p - v)/\tau_D,\ S_{wp} = \theta(w_p - w)/\tau_D,$$

$$S_{kp} = -2k\theta/\tau_D^*,\quad \tau_D = \rho_p d_p^2/18\mu_l f_D,\quad f_D = \frac{1 + 0.15\,\mathrm{Re}_p^{2/3}}{1 + 3.82A},\quad A = \mu_l/\rho c d_p,$$

$$\tau_D^* = \tau_D\left(1 - \mathrm{Re}_p \frac{\partial \ln \tau_D}{\partial \mathrm{Re}_p}\right)^{-1}.$$

Here, γ, u_p, v_p, w_p are the volume concentration and the components of the velocity of the disperse phase; τ_D, dynamic relaxation time of a particle, with a correction for the non-Stokes regime of flow about the particle f_D; Re_p, Reynolds number for the particle; c, sonic velocity; d_p, particle diameter; ρ_p, density of the particle material; $\theta = \rho_p\gamma$, mass concentration of the disperse phase. Equations of the Eulerian type [25] are written for the disperse phase:

$$\frac{\partial \gamma u_p r}{\partial z} + \frac{\partial \gamma v_p r}{\partial r} = 0,\quad v_p \frac{\partial v_p}{\partial r} + u_p \frac{\partial v_p}{\partial z} - \frac{w_p^2}{r} = \frac{1}{\tau_D}(v - v_p),$$
$$v_p \frac{\partial w_p}{\partial r} + u_p \frac{\partial w_p}{\partial z} + \frac{v_p w_p}{r} = \frac{1}{\tau_D}(w - w_p),\quad v_p \frac{\partial u_p}{\partial r} + u_p \frac{\partial u_p}{\partial z} = \frac{1}{\tau_D}(u - u_p). \tag{2.1}$$

The diffusion of particles can be ignored for the disperse-phase parameters being investigated. System (2.1) was solved in a unique algorithm with equations for the carrier phase. Figures 4 and 5 show the transverse profiles of u, u_p, w, w_p in the case of a dispersed-gas flow in a cylindrical channel when particles with $d_p = 2 \cdot 10^{-5}$ m and $\rho_p = 8.9 \cdot 10^3$ kg/m³ are chosen as the disperse phase and the ratio of the mass flows of particles and air at the channel inlet $\kappa = 1$. The initial profiles for the carrier phase were determined as follows: $u_0(r) = 100$ m/sec., $v_0(r) = 0$, $w_0(r) = 300\ r/R$ (m/sec), $R = 10$ cm is the channel radius. The following conditions were prescribed at the inlet for the disperse phase: $u_{p0}(r) = 10$ m/sec, $w_{p0}(r) = v_{p0}(r) = 0$. The profile for γ was uniform. As shown in Fig. 4, introduction of the particles decreases the nonuniformity of the profile for u, while the longitudinal dimension of the

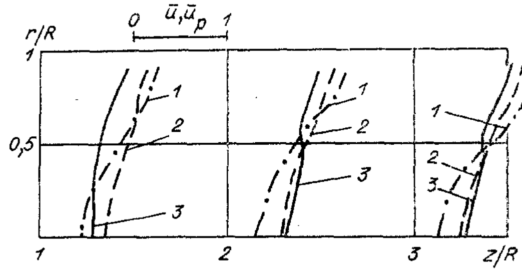

Fig. 4

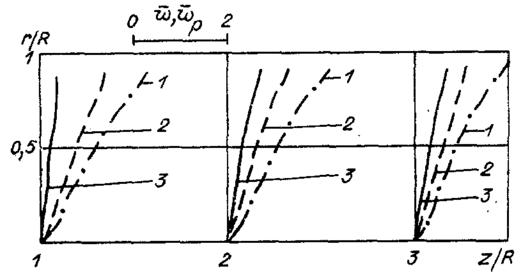

Fig. 5

profile in flows with a recirculation zone. The greatest difference for the velocities u and u_p is seen near the channel wall (curve 1 – profile of u in the case of a nondust-laden flow; 2 – profile of u in a dispersed-gas flow; 3 – profile of u_p; the notation for w and w_p in Fig. 5 is similar).

The presence of initially unswirling inert particles leads to the suppression of swirling (Fig. 5), although the profiles of w and w_p are qualitatively the same as for the case of rotation in accordance with the law of a solid. Thus, calculations performed using the proposed model showed that the disperse phase has a strong effect on the characteristics of the flow, which can be described within the framework of a simple modification of the k–ε turbulence model.

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